National Curriculum Programme of Study:

- Find 1000 more or less than a given number
- Estimate and use inverse operations to check answers to a calculation
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- Estimate, compare and calculate different measures, including money in pounds and pence

Y4

MENTAL CALCULATION

Addition & Subtraction

FLUENCY

By the end of Year 4, pupils should fluently derive and recall:

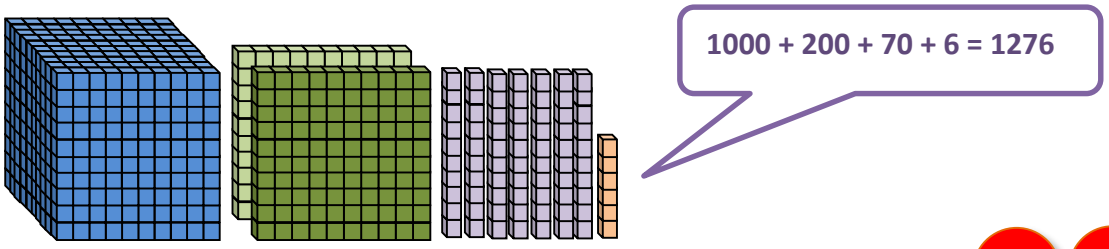
- the total of any two 2-digit number by partitioning or counting on
- all number bonds to 100 and to £1.00
- add to the next 100, £1 and whole number (234+66, 3.4+0.6, 81p+19p)
- sums and differences of pairs of multiples and near multiples of 10, 100 and 1000
- addition doubles of numbers 1 to 100

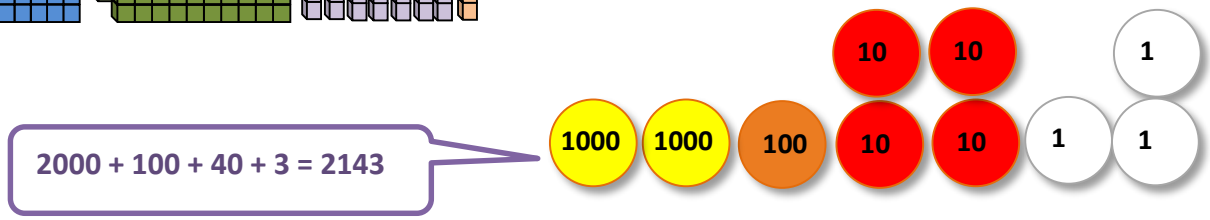
FIND 1000 MORE OR LESS THAN A GIVEN NUMBER

Teaching should focus on:

- Identifying the digits that will 'change' when a number is added/subtracted and reasoning why
- Providing clear visual models to support conceptual understanding of addition and subtraction
- Consolidating understanding of finding 1, 10, and 100 more or less than a given number

In conjunction with other place value activities, ensure the pupils have considerable opportunities to represent given numbers with up to four digits. They should be confident to use a range of resources and be able to explain how these can be partitioned in different ways, retaining the given value.





See also Bright Pi Mental Calculation Guidance; Year 3 Addition & Subtraction 'add and subtract numbers mentally including;

- three-digit number and ones
- three-digit number and tens
- three-digit number and hundreds'

This details activities and ideas for clearly modelling the addition of ones, tens and hundreds. Many of the models and ideas can be applied to the addition and/or subtraction of 1000 for Year 4.

Provide pupils with an individual 'Gattegno' (place value) chart, showing numbers up to 10 000 and then 100 000 when appropriate.
On an enlarged version of the chart cover numbers from each row to make a 4-digit or 5-digit number. Ask the pupils to use counters to cover the same numbers on their charts. What number do they have altogether? (48 263). How could we show adding 300? (move the green 'hundreds' counter three places to the right) Now we have 48 563.
Which counters need to move if we need to show subtracting four thousand? (blue). Move the counter four places to the left to model the subtraction.

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000
10000	20000	30000	40000	50000	60000	70000	80000	90000

Challenge the children to represent numbers in different ways, using their knowledge of partitioning. What number is represented here?(55 807)

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000
10000	20000	30000	40000	50000	60000	70000	80000	90000

Show the pupils an image of a milometer and discuss its purpose in a car or on a motorbike. Have the pupils ever watched the numbers change? Discuss how it works; like a digital abacus.

This milometer is showing 37 610 miles. When the car has travelled ten more miles, it will read 37 620. What will it read after a journey of 20, 50, 73 miles? Encourage the pupils to state the mileage as a 'real' number, e.g. "*thirty seven thousand, six hundred and eighty three miles"*. How many miles does the car need to travel for the milometer to show a distance ending with three zeros? (38 000 miles?

This milometer is showing 48 151.6 miles. Discuss what the *'point six'* means in terms of distance travelled. (Pupils will have been introduced to tenths in Year 3 and should be fluent in counting up and down in tenths). Ignoring the 0.6 miles, what will the mileage be if the car travels a distance of 1000 miles? 200 miles? 2009 miles?

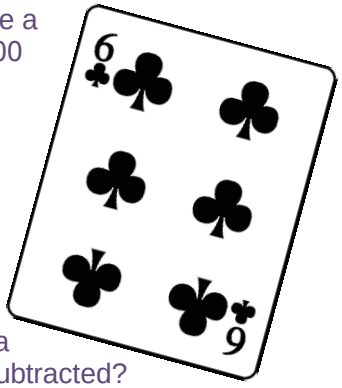
Provide other 'real life' contexts where multiples of 1000 need to be added or subtracted from large numbers.

e.g. Fred has a bank statement showing his account balance of £3 087. Each month he earns £2 000 from his job. Assuming he doesn't spend anything, and no other money is credited to his account, how long will it be before his balance reaches £10 000?

Contexts linked to measures can also be used, as several use units of 1000 (1000ml = 1litre, 1000m = 1km etc.)

Provide pupils with 'Ace' to '9' of each suit from a pack of cards (or 4 sets of 1-9 digit cards). Shuffle and place them face down. Take the top 4 cards and arrange them to make a 4 digit number, e.g. 7643.

Take a fifth card from the pile to give a 'thousands number', e.g. 6000



The multiple of 1000 can be added or subtracted to/from the four-digit number to give a final answer.

E.g. 7643 + 6000 = 13 643
or 7643 – 6000 = 1643

The aim is to get an answer as close to 5000 as possible.
In this example shown here, could the first four cards be arranged in a different way to get a final answer closer to 5000 once the 6000 is added or subtracted?

Pairs of children can challenge each other, the nearest to the 5000 target winning a point each time.

ESTIMATE AND USE INVERSE OPERATIONS TO CHECK ANSWERS TO A CALCULATION

Teaching should focus on:

- Ensuring pupils have a good understanding of the value of numbers to 10 000 and be able to partition them in different ways
- Positioning numbers on a number line and visually understanding rounding as the closest 10, 100, 1000, 10p, £1, £10 etc.
- Adding and subtracting multiples of 10, 100, 1000 to find approximate answers to calculations
- Using practical equipment to model the addition/subtraction inverse when calculating

Estimation should be used when calculating both mentally and when using a formal vertical written method. Encourage the pupils to ask themselves these questions…

Am I expecting my answer to be larger or smaller than the numbers I started with? Why?

Have I already carried out a similar calculation that would help me to know if my answer is reasonable?

Can I round the numbers in my calculation to give me an easier calculation that would be a good estimate for my actual calculation?

Once the calculation has been carried out, pupils should then be encouraged to compare their answer to their estimate – is it what they expected? Can they now use the inverse operation to 'work backwards' and check their working? These skills need to be explicitly taught and modelled carefully.

Provide children with a range of calculations already completed, and ask them to decide how they will check each one. Are some incorrect calculations easier to spot than others – why?

A	B	C
3652 + 7264 = 10 914	2520 – 1340 = 1220	503 + 2516 – 203 = 3222

D	E	F
2236 + 3101 = 5067	2357+ 2875 = 5233	8267 – 2319 = 5948

Encourage them to use their range of skills, identifying which are most useful. Skills might include;

- knowledge of sums of odd and even numbers (e.g. odd + odd = even, so 'E' cannot be correct)
- checking the 'ones' digits of each number being added (e.g. 2+4=6, so 'A' cannot be correct)
- rearranging numbers to make calculations easier (e.g. 'C' can be rearranged as 503 – 203 + 2516, so 300 + 2516 = 2816)
- rounding to the nearest 10, 100 or 100 as appropriate before adding/subtracting to reach an approximate answer (e.g. the numbers in 'D' can be rounded down to 2200 + 3100 = 5300. The exact answer should be greater than this, so 5067 cannot be correct)

See Bright Pi Mental Calculation Guidance; Year 3 Addition & Subtraction for additional ideas on developing estimation and checking skills.

SOLVE ADDITION AND SUBTRACTION TWO-STEP PROBLEMS IN CONTEXTS, DECIDING WHICH OPERATIONS AND METHODS TO USE AND WHY

Teaching should focus on:

- using and applying their skills of counting, reordering, compensating and knowledge of near doubles
- counting on or back in ones, tens, hundreds and thousands
- using brackets to accurately record multi-step calculations
- knowing when to 'take away' and when to 'find the difference' when subtracting (***See Year 1 Mental Calculation Guidance for further detail***)
- use partitioning when counting on or back in seconds, minutes or hours, bridging through 60 to solve problems in the context of time
- pupils understanding a given problem and explaining their reasoning in words, using practical equipment and number lines to model their mental calculation strategies

Provide a range of problems and activities, in different contexts, for pupils to practice their taught skills;

Consecutive Numbers

Introduce the children to consecutive numbers as 2 whole numbers that appear next to each other on the number line. Examine a 600-800 no. line.

Try some investigations;

- Add any two consecutive numbers. What do you notice?
- Find two consecutive numbers that total a number ending in 1. How many sets can you find? What do you notice?
- Add 2 consecutive numbers, then add the next two. What is the difference between the totals? Does this always happen? Why?
- Add 3 consecutive numbers. Can you predict whether the answer will be odd or even?
- Add pairs of consecutive odd numbers, e.g. 647 and 649. Look at the number between your umbers, i.e. 648. How is that connected to your answer? Does this always happen? What about pairs of consecutive even numbers? Why?
- What would happen if our number line contained both 2 and 3-digit numbers, e.g. from 85 to 145?
- Do the same patterns apply to four digit numbers? How do you know?

Develop a 'I wonder what would happen if…?' attitude to this style of investigation, encouraging pupils to challenge each other, think of new questions to explore and apply their mental calculation skills.
Pupils may need to be reminded to work systematically in order to help them spot any patterns. Separate calculations can be written on small pieces of paper or 'sticky notes', and then re-ordered to make pattern spotting, and thus generalisations, easier.

'Add This, Take That'

Pupils sit in a circle (as a whole class or in smaller groups), and start the count at a given 3-digit number. The teacher should announce an '*Add This, Take That'* rule, such as '*Add 400, Take 90'*. Pupils then count round the circle following the rule,

E.g. Start at 245… 555… 865…

Change the rule as appropriate; e.g. '*Add 50, Take 110'*, starting the count at 1536… 1476… 1416…

It is important for the pupils to visualise the numbers in the sequence, so the use of practical base 10 equipment or jottings will be crucial. Ask them to find efficient ways of calculating the steps given, e.g. for adding 90, they could add 100 and then subtract 10. Instead of adding 400 and taking 90, they could add 310.

'Camping Trip'

Provide a range of camping items available to buy from '*Come Outdoors'*. Pupils should decide what they would like to purchase for their trip, and use their mental addition skills to keep a running total as they add items to their list. Now provide a budget total, e.g. £700. Who has gone 'over budget'? How much over? How much money do other pupils have left? What can they buy with it?

Encourage children to keep jottings of the mental addition and subtraction calculations they have carried out. Ask them to note down any key skills they have used such as doubling, rounding, partitioning etc. If they replaced one item for another, e.g. swap their torch for a larger lamp, how much would it add to their bill? How do they know?

Hop, Skip, Jump & Leap

Tell the pupils that a 'hop' is worth 1, a 'skip' is worth 10, a 'jump' is worth 100 and a 'leap' is worth 1000. *I start on 372 and take two hops, a jump and two leaps forwards – what number do I land on?'* (372 + 2 + 100 + 2000 = 2474).

'I start on the number 3206 and take 3 'skips' forwards followed by a backwards 'leap'. What number do I finish on?' (3206 + 30 – 1000 = 2236).

'*I need to travel from 4510 to 2356. What are the fewest moves I can make?' Can I get to 2356 in exactly 6 moves?'*

'Choose 4 single digits and arrange them into a 4 digit number, e.g. 5368. Find the fewest number of moves needed to reach the reverse of your number, i.e. 8635. Investigate this further with other numbers; is it always the same number of moves?'

Provide the pupils with blank number lines to enable them to record their hops, skips, jumps and leaps. Encourage them to use a different colour for each type of move.

Introduce the rule that you cannot go 'past' your target, e.g. to get from 102 to 201, you cannot add 100 then subtract 1. Does this always increase the number of moves needed?

And so on…

Start with 1. Add 2, add 3, add 4, and so on…

Start with 1. Add 3, add 5 , add 7, and so on…

Start with 10. Add 3, add 5, add 7, and so on…

Start with 50. Add 3, add 4, add 3, add 4, and so on…

Start with 1. Add 7, subtract 3, add 7, subtract 3, and so on…

Start with 100. Subtract 1, then 2, then 3, then 4, and so on…

These 'and so on…' challenges are structured to give them more point and interest than simply completing a set of miscellaneous mental calculations. The structure may also give rise to other characteristics that may stimulate some investigation. E.g. starting with 1 and adding successive odd numbers ***produces*** *the square numbers.*

Challenge pupils to go as far as they can without any jottings, then add jottings when necessary. The subtractions may stop naturally when there appear to be not enough left to subtract from, but this is also an ideal context in which to develop understanding of negative numbers

'Spin for the Price'

In the 'Spin for the Price' shop all of the items are labelled with a number. Customers then spin the special shop wheel to determine the price they need to pay (in pence) Provide children with a list of items with numbers between 50 and 100. Ask them to choose an item and spin the wheel – what is the easiest way of calculating the price?

E.g. a football is labelled '65' and I spin the red section on the right of the wheel.

(480 – 65) + 25 = 440

So the football would cost £4.40

Take the opportunity to remind the pupils when and how to use brackets when writing down their multi-step calculations. Encourage them to share their calculation strategies for each of their items. Is there a quicker way? What mental calculation skills have they made use of?

Change the rules so that whatever colour they spin, they can choose which of the sectors of that colour they use for their calculation. How will they know which to choose? Why? Do some sectors always work out cheaper than others? How do they know?

Fancy a coffee?

Provide a price list for drinks and snacks in a café as well as ticket prices for the bus and train to a local town centre. Explain that they are to travel by either bus or train to meet their friend at the café, where they will pay for them each to have a drink and snack. They have just £10 to spend. How will they spend their money? How much will their café bill be? How much will they spend on travel? How much will they have left when they get home?

Encourage pupils to make decisions, try out ideas and jot them down. They shouldn't be concerned about committing to paper.

They might benefit from a final table to complete showing their purchases and calculations.

Ask them to think of further questions that could be asked within this context. "I wonder what would happen if I had £12 instead of 10?... if… if…"

ESTIMATE, COMPARE AND CALCULATE DIFFERENT MEASURES, INCLUDING MONEY IN POUNDS AND PENCE

Teaching should focus on:

- Providing clear visual models and images to support children in making estimates
- Consolidation of rounding skills for different measures, e.g. rounding to the nearest minute, hour, kilogram, metre, pound etc.
- Converting between different measures, e.g. kilometres to metres, hours to minutes etc. using knowledge of multiplication and division

It is commonly the decimal format of measurements that causes most difficulty for pupils when calculating. In the same way that pupils will have previously used number grids to support addition and subtraction of whole numbers, adapted grids can be effective in securing a mental image to add and subtract decimal numbers

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0

E.g. 3.5 + 2.3 = 5.8

E.g. 8.3 – 1.9 = 8.3 – 2.0 + 0.1 = 6.4

'Length' will be the most familiar measure to many pupils, and so is a useful starting point when encouraging them to use their mental calculation skills in different contexts.

Ask pupils to estimate how many times bigger the circumference of their head is than the span of their hand. Ask them to then cut two paper strips; one to match their hand span and the other to match their head circumference. Compare the strips to give an approximate answer (it is usually 3).

Encourage them to estimate and compare their length measurements by asking a series of other questions;

- If all the class handspans were put in a line, would it reach across the classroom? How do you know? How can we use approximation to find out?
- What is the difference between the largest and smallest handspans in the class?
- If all the head circumferences were added together, what would the total be to the nearest metre? Half metre?
- If that was the circumference of the head of a giant, approximately how big would his hands be? How tall would he be? How could we find out?

Other useful activities could be comparing length of reach and stride, circumference of waist and lower leg, height and foot length.

Similar activities can be carried out with mass (weight) with pupils can estimating and comparing weights of parcels to be loaded onto sack trucks with a maximum load displayed in kg. Capacity work can be linked to contexts such as cocktail making, adding quantities of ingredients to fit into a 2 litre bottle.

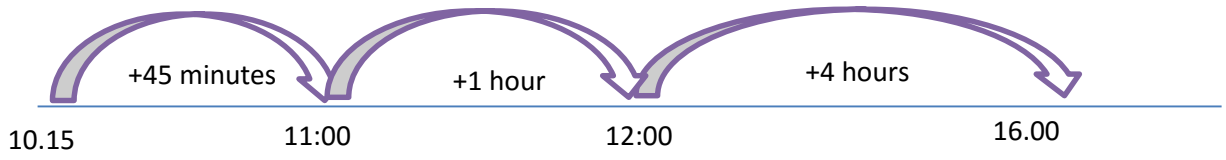
When calculating with time, it is important for pupils to understand that conventional vertical written methods are not appropriate due to the fact that time is measured in base 60, and not base 10. ***See Bright Pi Mental Calculation Guidance; Year 3 Addition & Subtraction 'Compare duration of events'*** for guidance on the use of mental addition and subtraction skills within the context of time.

Encourage the use of bridging skills on a time (number) line to support mental calculation, and combine statistics table-reading skills when solving problems in a time context.

E.g. coaches arrive and leave Poole at these times;

	Arrive	Leave
Coach A	08.00	14.30
Coach B	09.30	15.45
Coach C	10.15	16.00
Coach D	11.45	17.30

Which coach gives the most time in Poole? Which gives you the least time?
E.g. Coach C might be modelled as…



	Barnstaple	Bristol	Exeter	Penzance	Plymouth	Taunton
Barnstaple						
Bristol	100					
Exeter	55	84				
Penzance	108	194	110			
Plymouth	67	125	44	77		
Taunton	50	51	34	144	75	

Show a mileage chart between towns and cities and allow pupils time to discuss the layout and how it could be used. Ask simple questions to familiarise them with the layout; *"How far is it from Taunton to Exeter?" "How much further is it from Bristol to Plymouth than from Penzance to Plymouth?"*

Ask them to recreate the chart, rounding each distance to the nearest 10 miles. Answer the same questions as before, using the new chart. How will their answers differ to before? Why? Is this as useful to a driver planning their journey?

If I travelled from home in Penzance to Taunton on Monday, then on to Bristol on Tuesday, how far will I have travelled in total? At the start of my journey the milometer on my car displayed '0036589' – what will the display read once I reach Bristol? The display in my car shows that I have enough fuel to travel a further 110 miles. Can I get home without running out of fuel?

By this stage, pupils will be carrying out formal vertical addition and subtraction calculations, with decimal measures such as money as well as with whole numbers. These written methods should not replace the use of key mental skills when working within these contexts.

See Bright Pi Mental Calculation Guidance; Year 3 Addition & Subtraction 'Add and subtract amounts of money to give change' for guidance on the use of number lines as a visual model when working in a money context.