National Curriculum Programme of Study:

- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract number using concrete objects, pictorial representations and mentally, including;
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- Solve problems with addition & subtraction, using concrete objects and pictorial representations, applying their increasing knowledge of mental and written methods

Y2

MENTAL CALCULATION
Addition & Subtraction

FLUENCY

By the end of Year 2, pupils should fluently derive and recall:

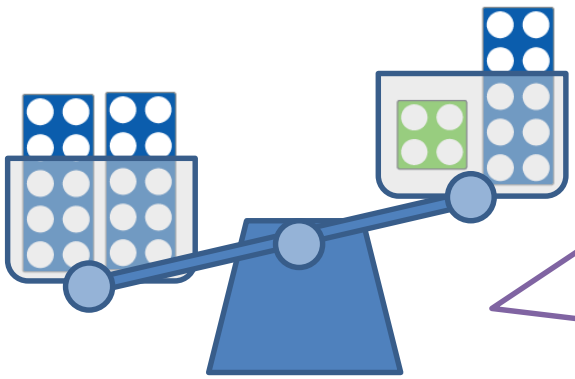
- addition and subtraction facts for all numbers up to at least 20, e.g. 3 + 4, 18 – 5
- number pairs with totals to 20, e.g. 17 + 3, or what to add to a 'teens' number to make 20, e.g. ☐ + 13 = 20
- all pairs of multiples of 10 with totals up to 100, e.g. 20+80, or 30+☐=100
- what must be added to any two-digit number to make the next multiple of 10, e.g. 32+☐=40
- addition doubles for all numbers to at least 20, e.g. 18 + 18, and multiples of 10 to 50, e.g. 40+40

RECALL AND USE ADDITION AND SUBTRACTION FACTS TO 20 FLUENTLY, AND DERIVE AND USE RELATED FACTS UP TO 100

Teaching should focus on:

- Practise and consolidation of addition and subtraction number facts to 10 and 20 (and all numbers up to 20), increasing in recall speed. E.g. 12 + 6, 19 – 5, 12 + ☐ etc.
- Providing a range of visual images for the pupils to understand how facts are related and pupils drawing their own

- Some practical mathematics resources are weighted according to their value (e.g. *Cuisenaire* number rods or *Numicon* shapes). If these are available, then they can be used with a balance scale to consolidate understanding of facts to 20.



The left side is heavier so it is greater in value, the number is larger. How many do we have on the left? (20) On the right? (14)

What Numicon shapes could I add to the right 'pan' to get the scales to balance? What if I only had 'number 2' pieces?

Could I make it balance by adding exactly three different pieces? Four? Could I add seven pieces? Why not?

Encourage pupils to write number sentences alongside their visual models. In the starting example shown above, initial number sentences might include;

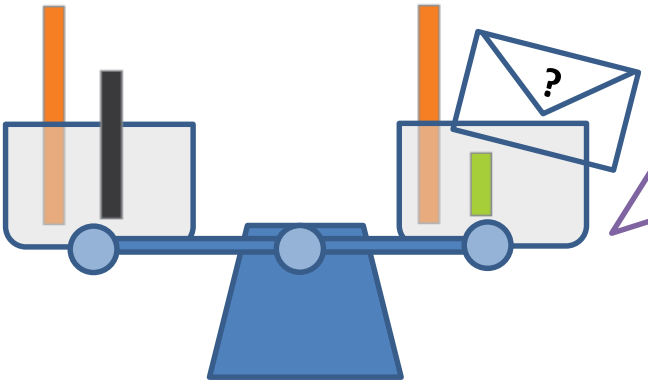
'20 is more than 14' '14 is less than 20' '14<20' '20>14' '20 is not equal to 14'

They can be challenged to write other number sentences once they have 'balanced' their scales;

'10 + 10 = 4 + 10 + 6' '20 = 4 + 10 + 6' '20 = 14 + 2 + 2 + 2' (adding only '2' pieces)

'20 = 14 + 3 + 2 + 1' (adding three different pieces) '20 = 14 + 1 + 1 + 2 + 2' (adding four pieces)

The use of pan balances in this way supports a firm conceptual understanding of the equality sign (=) as a symbol of 'balance' and not 'answer'. An envelope containing a mystery piece of equipment (*Cuisenaire* number rod in the image below) makes the concept of 'missing number' more concrete for the pupils. This can the lead on to other 'missing number' work, relating back to practical experiences.



10 + 7 = 17

10 + 3 + ? = 15

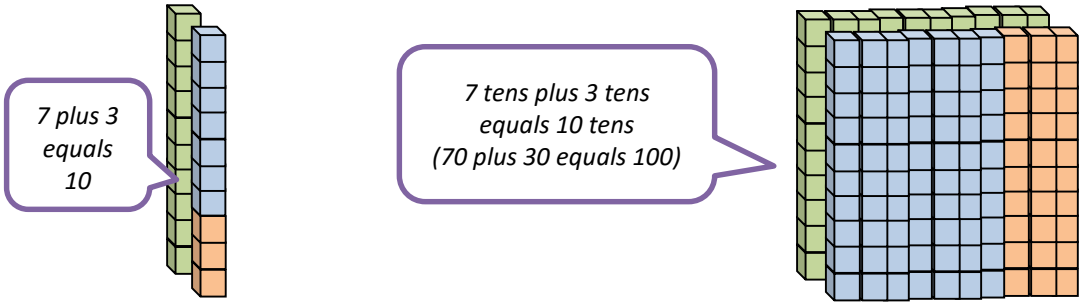
The scales are exactly balanced so the numbers on either side must be equal. What number is shown on the left side? (17)

What can you see on the right side? (10 and 3 and an envelope) What is the total of the number rods you can see? (13) We need to find out what other rod is in the envelope. Could it be another 10 rod? Why not? What could it be? How do you know?

Could there be two rods in the envelope? What could they be?

If weighted resources are not available, then similar activities can be set up 'virtually'

- Practical equipment grouped in tens (Dienes, straw bundles) can provide a useful visual support for pupils relating facts they already know to those they don't. E.g. A pupil who knows 3+7=10 can also see that 7+3=10, 10-3=7, 10-7=3. Ask them to then choose the same digits, but to select equipment ten times bigger. E.g. 30 and 70. Links can be made and they can then calculate 30+70=100, 100-70=30 and 70=100-30 and answer missing number questions based on known facts. 10 = 3 + ? 100=30+?



- This can be linked to work on a *Gattegno* chart such as that shown below. Pupils start on the bottom line of single digit numbers and shade two small numbers (less than 5), circling their total.
E.g.

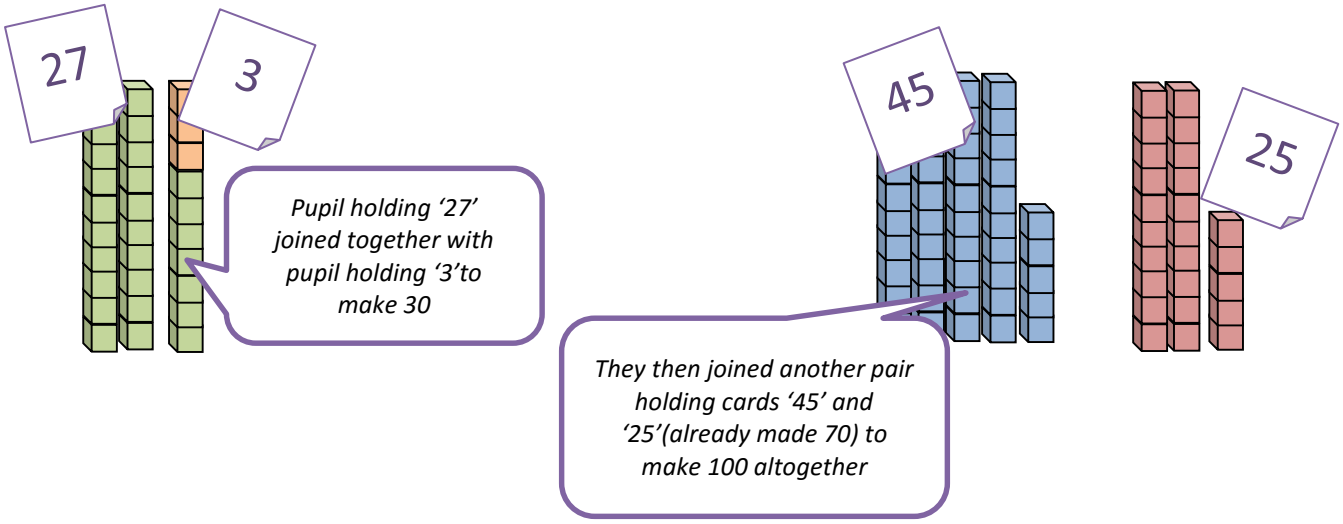
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Now ask them to move to the middle row, where the numbers are ten times bigger. Ask them to shade the same numbers as before, but ten times bigger. What do they notice when they add them? Does this work for the top row too? Can they prove it using equipment?

See also Mental Calculation Guidance
Year 2 Multiplication & Division *'patterns in the ten times table'*

- 'Stand Up, Pair Up'; give each pupil in the room a number card and give them 10 seconds to find a partner whose number will add to theirs to make a multiple of 10. Will their partner need to be odd or even? Why? In their new pair, now find another pair who they could join for a total of 100. How do they know who look for? What number fact knowledge are they using?

The teacher could model a representation of one group's numbers, using practical equipment;



- '*Have they all gone?*' (*adapted from 'Strike It Out', Nrich)*
  Provide each pair of pupils with a number line, marked from 0 to 20. Ask them to decide who will be player A and who will be B. Explain the rules of *'Have they all gone?*'…

  Player A starts by crossing out two numbers and deciding whether to add them or find their difference. The answer must be on the number line. They draw a circle around their answer (4 + 9 = 13 shown here in red)

  Player B (marked in blue) starts by crossing out the circled number, choosing another to add or find the difference, and circle their answer (13 – 10 = 3). Each number may only be used once. Have all the numbers gone? How many numbers were not used? The rules can be adapted to suit the pupils…
- Players A & B are 'against' each other, and try to choose numbers so that there are no options left for the other player to have their turn
- Players A & B play together as a team, trying to choose numbers so that they can cross off as many as possible

  hallenge the pupils with additional questions… *Is it possible to cross off all of the numbers? How? Why not? Is it possible to use only addition? Only subtraction? What if you could also use multiplication and division? If the line had only even numbers, say 2 to 40, would the game be easier or harder?*

ADD AND SUBTRACT NUMBERS USING CONCRETE OBJECTS, PICTORIAL REPRESENTATIONS AND MENTALLY, INCLUDING;

- 2-DIGIT NUMBER AND ONES
- 2-DIGIT NUMBER AND TENS
- TWO 2-DIGIT NUMBERS
- THREE 1-DIGIT NUMBERS

Teaching should focus on:

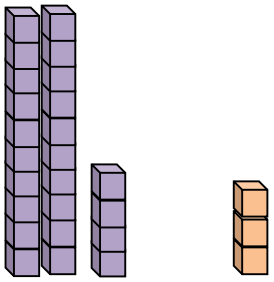
- Quick recall of number bonds to 10 and 20
- Securing an understanding of 'take away' and 'finding the difference' and how they differ
- Counting on and back in ones from any two-digit number, e.g. 55+3, 45-3 …
- Counting on and back in tens, from any two-digit number, e.g. 47+30, 63-40…
- Compensation skills when adding/subtracting, e.g. 36+19 (add 20 then subtract 1), 57-29 (subtract 30, then add 1)
- Identifying the nearest multiples of 10 to a given number to support bridging when calculating

See also Bright Pi Written Calculation Guidance;
Year 2 Addition *'using grouped objects for addition without regrouping and with regrouping'*
Year 2 Subtraction *'using grouped objects for subtraction without exchanging and with exchanging'*

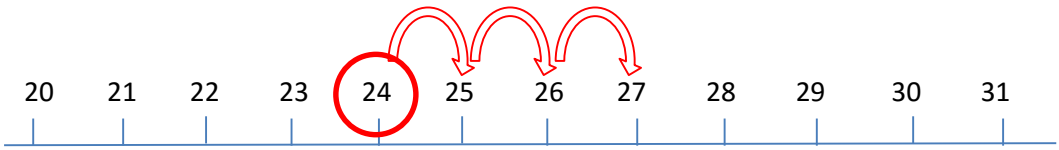
Note
There is an important progression from adding/subtracting a 2-digit number and ones (e.g. 24+5, 36-8), to adding/subtracting a 2-digit number and tens (e.g. 24+30, 67-30) to adding/subtracting two 2-digit numbers (e.g. 24+32, 54-33)

- **Adding/subtracting a 2-digit number and ones**
  Pupils need to be exposed to a range of mathematical resources, visual images and models in order to secure a good understanding of the structure of the number system. Number lines and grids used in a range of ways can support them in looking for patterns and calculating with small numbers.

  A number line is a useful tool for recording and supporting mental calculations, and might accompany practical work once pupils are working confidently with the concrete materials.
  E.g.

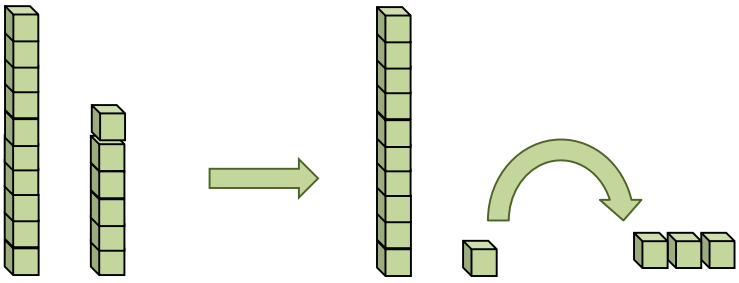


24 add 3…
Start by 'making' 24, then add three more, counting the total as the blocks are added; *"24…25…26…27 The total is 27"*



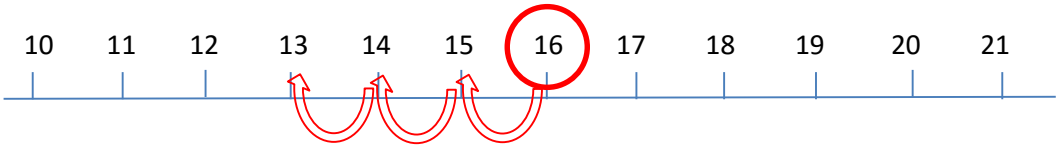
As when using the practical apparatus, pupils should usually be encouraged to reorder the numbers when adding on a number line, to start with the number of greatest value. Ideally they should learn this through experience – ask them to try a calculation twice, starting with each of the numbers. Which is quickest and easiest? Why?

When subtracting a small single-digit number from a larger number, it is usually more efficient to use the concept of 'take away' rather than 'find the difference' ***See Mental Calculation Guidance Year 1, Addition & Subtraction*** *for further definition.*



"16 take away 3…"
Start by 'making' 16, then take away one cube at a time, counting the number of blocks removed *"1..2..3."* Finally count the number of cubes remaining, *"10 and 1 is 11, there are 11 cubes left. 16 take away 3 is 11"*

As pupils become more confident counting backwards from any number, encourage them to keep count of the remaining cubes as they 'take away', e.g. *"16…15…14…13 There are 13 cubes left"*



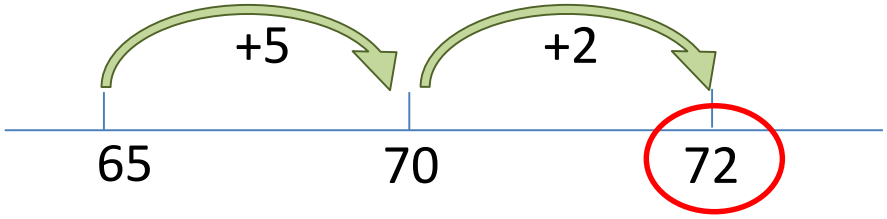
It is important that number lines increase from left to right, so when taking away, pupils should be encouraged to find their starting number, and 'jump back' the required number of 'ones'. The arrows can be above or below the line (arrows are shown here below the line to distinguish subtraction from addition)

Ensure pupils have experience of adding/subtracting single digit numbers to/from 2-digit numbers in different contexts;

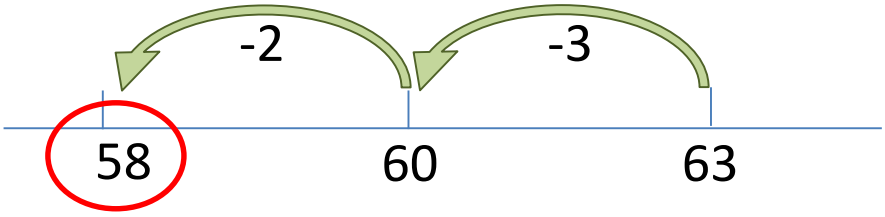
- Money: I have 84 pence when I go shopping. I spend 7 pence – how much do I have left?
- Length: My dad says I need to grow another 3cm before I'm allowed to go on the big ride at the fair. The sign says I need to be 96cm. How tall am I now?
- Time: It takes me 18 minutes to eat my breakfast in the morning, and then 4 minutes to clean my teeth. How long is that altogether? I then spend 8 minutes getting my school uniform on. How long does it take me to get completely ready for school?

Once pupils begin to gain an appreciation of number and how close a number is to the next or previous multiple of 10, it is often useful to count on or back in two steps (rather than in ones), bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks' is helpful, since pupils can visualise jumping onto them. It is still important to model with corresponding physical objects alongside the line.

E.g. 65+7 can be worked out in two jumps, first to 70, then to 72. The answer is the last point marked on the line, 13.



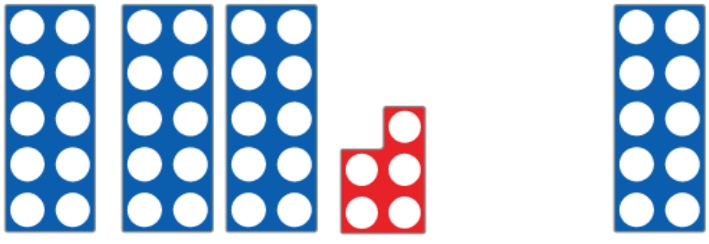
E.g. 63-5 can be worked out in two jumps by 'taking away' (as the numbers are relatively far apart) first 3 to 60, then the remaining 2. The answer is the last point marked on the line, 58.



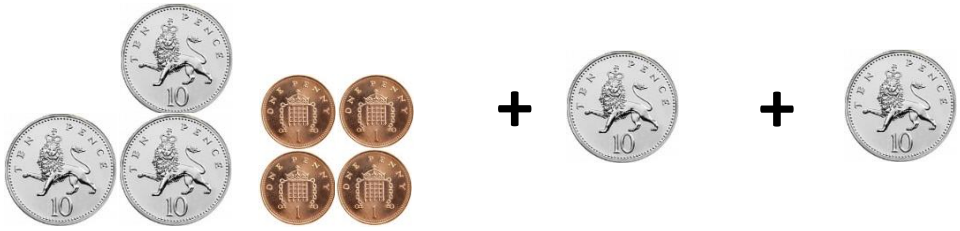
- **Adding/subtracting a 2-digit number and tens**

It is essential that pupils physically experience the concept of adding and subtracting tens with mathematical resources grouped in tens. Ask pupils to choose a two digit number (not a multiple of ten) and represent it with equipment of their choice using the smallest number of items as possible. This might be using straws, Dienes, Numicon, Multilink Cuisenaire rods, coins (10p/1p), masses (10g/1g) etc.

E.g. 35



Ask them to add ten to their number and explain the result. What changed? What stayed the same? What if we added another ten? And another? What happens if a ten is subtracted? Does the same part of the number stay the same as before?



Equipment can also be arranged into tens/ones columns using baseboards (***See Bright Pi Written Calculation Guide, Year 2 addition & subtraction)***

Number grids (1-100 or 0-99) are useful tools when adding/subtracting 10. Pupils should be encouraged to be 'Pattern Spotters' and to try to explain the patterns they can see. Where are the even numbers on the grid? The odd ones? Point out the mis-matched language when using number grids that involve moving 'down' the grid when counting 'up'.

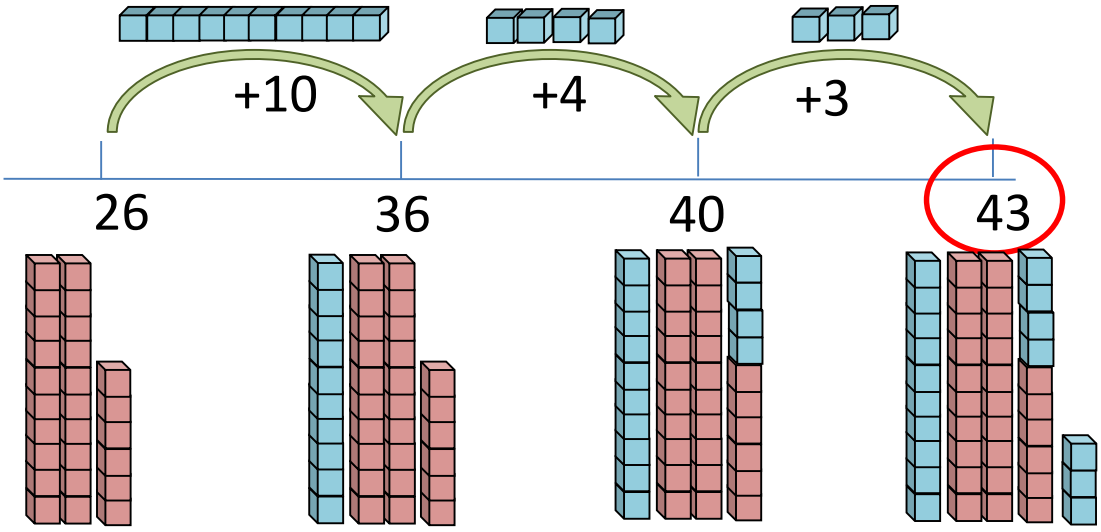
Play '*Where am I?*' using a 1-100 grid. Provide everyone with a small grid and ask them to position their finger on 24. Do the same on a large grid visible to all. Call out instructions and model how they should move their finger to match the instruction. E.g. *"add two" (move two numbers to the right), "add ten" (move 'down' one number), "take away 3" (move three numbers to the left), "add thirty" (move 'down' three tens) etc.*
Ask more able pupils to try to challenge themselves by imagining the grid in their head, only using their actual grid where necessary. Include the concept of inverse by asking questions such as "*I ended on 64 after adding three tens to my starting number. What number did I start on?"*

- **Adding/subtracting two 2-digit numbers**
A blank or partially marked number line can also be used to record 2-digit addition and subtraction calculations. Pupils still need to be encouraged to look carefully at the numbers and operation involved, and make decisions regarding the best way of working. They should be encouraged to 'pattern spot', look for 'short cuts' and discover helpful hints.

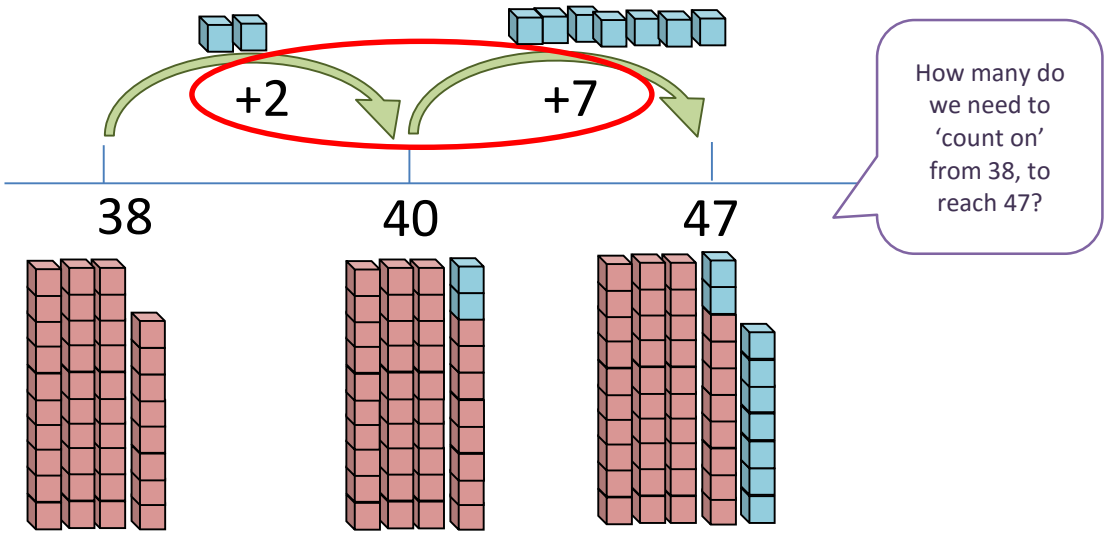
NOTE: The use of a number line is NOT a formal written method, it is simply a way of recording mental calculations. This might be to keep track of interim steps, or to concretely model to another person what has taken place mentally.

Being able to count on or back in two or more steps (rather than in ones), bridging a multiple of 10 is now crucial. If pupils struggle to identify the nearest multiple of 10 and the distance from it, they will resort to counting in ones.
In order to use the number line flexibly, pupils should be confident with adding multiples of 10, recognising distance from nearest multiples of 10 landmarks as well as partitioning numbers into tens and ones.

E.g. 26+17 can be found by starting with the largest number (26), adding the tens from the second number (10), bridging to the next multiple of 10 by adding 4, then adding the remaining 3. The answer is the last point marked on the line, 83.
Again model with corresponding concrete apparatus, e.g. 'make 26' using Dienes, add a ten rods, then add 4 ones cubes before adding the final 3 ones cubes. Using different colours helps to distinguish between the two numbers in the calculation and supports discussions around inverse. *"If we started with the pile of 43, and took 17 away (the blue ones), what would we have left?"*

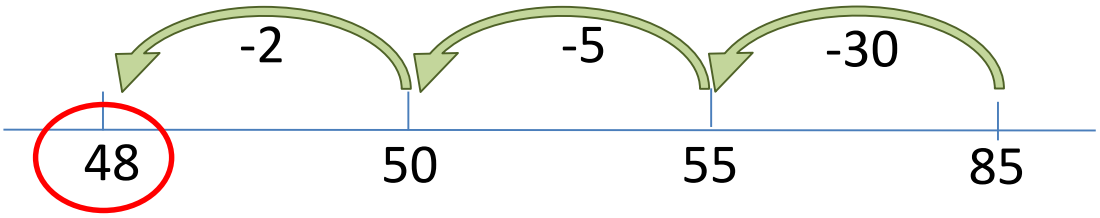


E.g. 47 – 38 can be found by 'counting up' from 38 to 47 (finding the difference as the numbers are relatively close together). The answer is the total distance between the two numbers 47 and 38, i.e. 9.

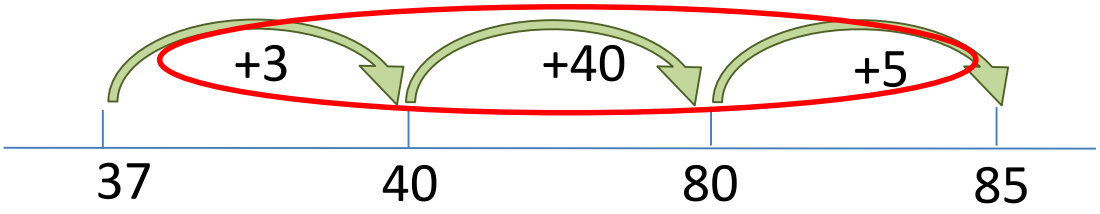


Sometimes it is not obvious whether to 'take away' or 'find the difference' (count up) and so both methods should be clearly modelled to the pupils, with similarities and differences noted.

Taking Away
85-37 can be found by taking away (partitioning 37 into 30,5 and 2). The answer is the last point marked on the line, 48.



Finding the Difference
85-37 can be found by 'counting up' from 37 to 85, first by adding 3 to reach 40, adding 40 to reach 80, then 5 to reach 85. The answer is the distance between the two numbers 85 and 37, i.e. 48



Ideally the use of number lines in this way should be modelled alongside the use of practical equipment. For a 'take away' model, build the first number using structured base-10 apparatus and physically take the relevant portion

Once pupils start to add/subtract two 2-digit numbers, they should start to work vertically with practical base-10 equipment (Dienes, Numicon, straws, Multilink towers etc.) on a base-board, preparing them for written work in columns in Key Stage 2.

See Bright Pi Written Calculation Guidance; Year 2 Addition and Year 2 Subtraction

Despite being introduced to working vertically with equipment, pupils still need to be encouraged to look carefully at the numbers and operation involved, and make decisions regarding the best way of working. They may need to draw a number line to support their mental calculation, or they may be able to use (or imagine) a 1-100 grid, adding/subtracting tens by moving vertically and adding/subtracting ones by moving horizontally. Skills of partitioning, compensation and doubling are now crucial.

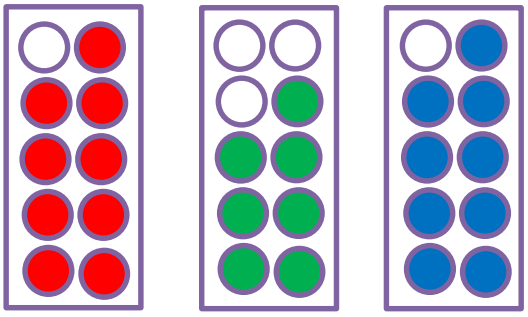
Compensation;
E.g. when adding 54 and 29, pupils should be taught to use the skills of 'compensation' by adding 30 onto 54, and then subtracting 1
E.g. 73-19, pupils should be encouraged to subtract 20, then 'compensate' by adding 1 back on

Doubling;
E.g. when adding 54 and 55, they should be taught to 'spot' the near double. They might double 55 and subtract 1, or double 54 and add 1

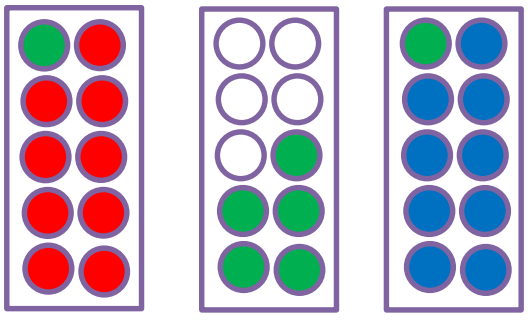
- **Adding/subtracting three 1-digit numbers**
Pupils need to be able to recall number bonds to 10 with relative speed in order to be able to add and/or subtract with three single-digit numbers.

Tens charts (array of 2 columns of 5, in an enclosed rectangle) are a useful visual tool when adding several single digit numbers together. Large egg boxes cut to size work perfectly in the classroom as pupils can drop objects into the 'holes' to represent their numbers. Alternatively, printed tens charts can be covered with small counters to represent different values.

"I have 9 red pens, 7 green pens and 9 blue pens. How many pens do I have altogether?"



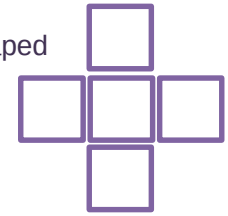
Represent the numbers of coloured pens in each tens chart. *"How could we find the total number of pens? Could we rearrange them in any way to make them easier to count?*



The 'pens' are rearranged to fill the tens charts. *What can you see now? Do we still have 9, 7 and 9? (yes, we have the same amount but it looks different) We can now see 10+5+10=25*
So 10+5+10 = 9+7+9
How many more to make 30?

Provide the pupils with a selection of numbers and ask them to choose three to add together. Which three can they add the quickest? Why? What strategies did they use? Did they make use of the tens charts, or imagine them?

Challenge pupils to position the digits 3, 4, 5, 6 & 7 into the spaces on this cross shaped diagram, so that the horizontal and vertical row equal the same total. What skills might they use? Are the tens charts helpful? Can they solve it with an odd number in the centre? An even number? What if they have digits 2-6 instead? Can they make up another 'challenge' using the same grid?



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SOLVE PROBLEMS WITH ADDITION & SUBTRACTION, USING CONCRETE OBJECTS AND PICTORIAL REPRESENTATIONS, APPLYING THEIR INCREASING KNOWLEDGE OF MENTAL AND WRITTEN METHODS

Teaching should focus on:

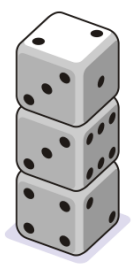
- Using and applying their skills of counting, reordering, compensating and knowledge of near doubles
- Pupils understanding a given problem and explaining their reasoning in words, using base 10 equipment and number lines to model their mental calculation strategies
- Deciding when it is appropriate to use practical equipment either informally or in formal columns on a baseboard, mental methods or informal jottings such as a number line.

Provide a range of problems and activities, in different contexts, for pupils to practice their taught skills;

The Funny Sweet Shop
Ask the pupils to recall the different coin denominations. Tell them about the 'Funny Sweet Shop' where the shopkeeper refuses to give change, and everything has to be paid for with exactly 2 coins!
E.g. the item might cost 12p, so you could pay with a 10p and a 2p coin.
What are all the possible prices for items in the shop? What is the highest price they can charge for any item? Can they sell a packet of sweets for 23p?

Ensure replica/real coins are available to support their thinking. Once they've worked out a few totals, encourage pupils to work systematically, starting with the cheapest or most expensive item. How might they record their work? Can they use sentence starters to support their explanations of how they found their solutions?

'Stack the Dice'
Stack three 1-6 dice on top of each other. What is the total of the front face on each die? Add the top two die and then subtract the bottom.
Think of the top two dice as a two digit number and subtract the bottom die.
What other numbers can we make by adding and subtracting them?



What strategies are the pupils using to add/subtract their numbers? How can they show their working on a number grid or line? Provide extra dice and ask the pupils to think of further calculations they could carry out. Provide a fourth dice. Split them into two 2-digit numbers and find their sum/difference. What is the largest number they can make?

'Alphabet Money'
Explain that the letter A costs 1p, B costs 2p, C costs 3p… etc. What is your first name worth? Is your full name odd or even? What is the value of the month of your birthday? What coins could you use? Can you find a word worth 25p? £1.00? Which number, in words, from zero to ten is worth the most?

Allow pupils to work in pairs to discuss a strategy – will they write out the whole alphabet with corresponding values before they begin? Ensure replica/real coins are available - do they allocate these to the letters?
What other questions can they think of to challenge the rest of the class?

'Add This, Take That'
Pupils sit in a circle (as a whole class or in smaller groups), and start the count at a given 2-digit number. The teacher should announce an '*Add This, Take That'* rule, such as '*Add 2, Take 1'*. Pupils then count round the circle following the rule,
E.g. Start at 15… 17… 16… 18…17…19…18…20…19…

When explaining the game initially, model the count on a number line, showing the jumps of 2 forwards (adding) and the jumps of 1 backwards (subtracting). Some pupils will benefit from having small number lines to annotate as the count continues. Can they see a pattern? Do they think they will say every multiple of 10 aloud? How do they know?

'Dart Board'
Use a dartboard like '*pin the tail on the donkey'* with pupils adding their score mentally, using equipment as necessary. Where would they like their darts to land? Explain the 'double' rule. What if I scored 5, 15 and double 20 and my friend scored all three darts in '20' – who has the highest score?

With a dartboard numbers in multiples of 5 and 10, base-10 equipment such as Dienes blocks, Numicon or straws are ideal for modelling. Show a final score in equipment and ask the pupils where the three darts could have landed.